Renormalization of the neutrino mass operators in the multi-Higgs-doublet standard model

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Abstract. We derive the renormalization group equations (RGE) for the flavor coupling matrices of the effective dimension-five operators which yield Majorana neutrino masses in the multi-Higgs-doublet standard model; in particular, we consider the case where two different scalar doublets occur in those operators. We also write down the RGE for the scalar-potential quartic couplings and for the Yukawa couplings of that model, in the absence of quarks. As an application of the RGE, we consider two models which, based on a μ–τ interchange symmetry, predict maximal atmospheric neutrino mixing, together with $U_{e3} = 0$, at the seesaw scale. We estimate the change of those predictions due to the evolution of the coupling matrices of the effective mass operators from the seesaw scale down to the electroweak scale. We derive an upper bound on that change, thereby finding that the radiative corrections to those predictions are in general negligible.

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1 Introduction

The standard model (SM) in the strict sense, i.e. without right-handed neutrino singlets, forbids neutrino masses. However, it was noticed a long time ago [1] that, if one allows for lepton-number non-conservation, then one can construct, with the SM multiplets, operators of dimension higher than four which give Majorana masses to the neutrinos. The lowest-dimensional such operators have dimension five and contain two left-handed lepton doublets and two Higgs doublets; those operators can be thought of as arising from the seesaw mechanism [2] after one has integrated out the right-handed neutrino singlets.[1] Under the assumption that the SM is valid up to the seesaw scale $m_{\rm R}$, the renormalization group evolution of the dimension-five neutrino mass operators from $m_{\rm R}$ down to the electroweak scale, as represented for instance by the Z^0 mass m_Z, can be determined; the evolution equations have been computed in the SM and in its minimal supersymmetric extension [4–7] (for a review see [8]). This is an important issue in view of testing mechanisms and symmetries for explaining the neutrino masses and the lepton mixing angles, since such mechanisms and symmetries are usually operative or imposed at the seesaw scale, while the measurements are effected at

the electroweak scale. (For the experimental and theoretical status of neutrino masses and lepton mixing see, for instance, [9] and [10], respectively.)

In this paper we extend the existing results for the SM renormalization group equations (RGE) to the case of an arbitrary number of Higgs doublets. In particular, we focus on dimension-five neutrino mass operators which contain two *different* Higgs doublets; indeed, to our knowledge, that case has not yet been treated in the literature. The reason to consider the multi-Higgs-doublet SM is that, within that framework, several models have been produced in recent years which predict, for instance, lepton mixing angles $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ [11, 12], or $\theta_{13} = 0$ alone [13, 14], or $\theta_{23} = \pi/4$ and $\delta = \pi/2$ [15].[2] (See [9, 10], for instance, for the definition of the lepton mixing angles.) Those predictions usually hold at the seesaw scale and, in order to compare them with experiment, one needs to know the corresponding corrections at the electroweak scale.

In Sect. 2 we display the Lagrangian of the multi-Higgs-doublet SM, without quarks but with dimension-five neutrino mass operators, and present the RGE for the couplings of that Lagrangian. In Sect. 3 we discuss the specific RGE for the models, referred to above, which predict $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ at the seesaw scale. In Sect. 4 we show explicitly how those seesaw-scale predictions arise, and how

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[1] The effect of the dimension-six operators which also arise from the seesaw mechanism has been studied in [3].

[2] The predictions $\theta_{23} = \pi/4$ and $\delta = \pi/2$ have first been obtained in a supersymmetric extension of the SM [16].

they may be changed by the renormalization group evolution. In Sect. 5 we derive an upper bound on the effect of that evolution. A short summary of our main results is provided in Sect. 6. An appendix contains some details of the calculation of the beta functions for the neutrino mass operators.

2 General case

2.1 The model

We consider the SM with n_H Higgs doublets ϕ_i ($i = 1, 2, \ldots, n_H$) with weak hypercharge 1/2. The $SU(2)$ gauge coupling constant is denoted g while the $U(1)$ gauge coupling constant (with the above normalization for the weak hypercharge) is denoted g'. The scalar potential V has the form

$$V = \text{quadratic terms} + \sum_{i,j,k,l=1}^{n_H} \lambda_{ijkl} \left(\phi_i^\dagger \phi_j \right) \left(\phi_k^\dagger \phi_l \right), \quad (1)$$

where the dimensionless couplings λ_{ijkl} satisfy

$$\lambda_{ijkl} = \lambda_{klij} = \lambda_{jilk}^*. \quad (2)$$

The lepton Yukawa Lagrangian $\mathcal{L}_{Y\ell}$ is

$$\mathcal{L}_{Y\ell} = -\sum_{i=1}^{n_H} \left(\bar{\ell}_R \phi_i^\dagger Y_i D_L + \bar{D}_L Y_i^\dagger \phi_i \ell_R \right), \quad (3)$$

where D_L denotes the left-handed lepton doublets and ℓ_R the right-handed charged-lepton singlets. We have defined the dimensionless flavor coupling matrices Y_i in the same way as [6,7]. Note that in this paper we do not use the summation convention.

The effective dimension-five neutrino mass operators are defined as

$$\mathcal{O}_{ij} = \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \left(D_{L\alpha a}^T \kappa_{\alpha\beta}^{(ij)} C^{-1} D_{L\beta c} \right)$$
$$\times \left(\varepsilon^{ab} \phi_{ib} \right) \left(\varepsilon^{cd} \phi_{jd} \right), \quad (4)$$

where, contrary to what we had done in (1) and (3), we have made explicit both the flavor and gauge-$SU(2)$ indices, and the summations thereover. In (4), C is the Dirac–Pauli charge-conjugation matrix; α and β are flavor indices; a, b, c, and d are $SU(2)$ indices; and ε is the antisymmetric 2×2 matrix, with $\epsilon^{12} = 1$. The flavor coupling matrices $\kappa^{(ij)}$ in (4) have dimension -1 and satisfy

$$\kappa_{\alpha\beta}^{(ij)} = \kappa_{\beta\alpha}^{(ji)}, \text{ i.e. } \kappa^{(ij)} = \kappa^{(ji)\,T}. \quad (5)$$

2.2 The RGE

The RGE are first-order differential equations which give the evolution of the couplings of a model relative to $t = \ln \mu$,

where μ is the mass parameter used in the regularization of ultraviolet-divergent integrals. We have computed, at the one-loop level, the RGE for the model outlined in the previous subsection. For the RGE of the coupling matrices of the effective mass operators we have found

$$16\pi^2 \frac{d\kappa^{(ij)}}{dt}$$

$$= -3g^2 \kappa^{(ij)} + 4 \sum_{k,l=1}^{n_H} \lambda_{kilj} \kappa^{(kl)}$$

$$+ \sum_{k=1}^{n_H} \left[T_{ki} \kappa^{(kj)} + T_{kj} \kappa^{(ik)} \right] + \kappa^{(ij)} P + P^T \kappa^{(ij)}$$

$$+ 2 \sum_{k=1}^{n_H} \left\{ \kappa^{(kj)} Y_i^\dagger Y_k - \left[\kappa^{(ik)} + \kappa^{(ki)} \right] Y_j^\dagger Y_k \right.$$

$$\left. + Y_k^T Y_j^* \kappa^{(ik)} - Y_k^T Y_i^* \left[\kappa^{(kj)} + \kappa^{(jk)} \right] \right\}, (6)$$

where

$$T_{ij} := \text{tr} \left(Y_i Y_j^\dagger \right), \quad (7)$$

$$P := \frac{1}{2} \sum_{k=1}^{n_H} Y_k^\dagger Y_k. \quad (8)$$

The last line of (6) is obtained from the previous line through the interchange $i \leftrightarrow j$ together with transposition, in agreement with (5). Our result (6) coincides, when $i = j$, with the result given in [6,7]; it generalizes that result for the case $i \neq j$. Note that, for the sake of simplicity, in the present paper we dismiss quarks; in general, one would have to add to (7) analogous trace terms featuring the Yukawa-coupling matrices of the Higgs doublets ϕ_i and ϕ_j to the up and down quarks, multiplied by a color factor 3 – see, for instance, [5]. Trace terms will be irrelevant for the discussion of the models in the next section.

The terms in the last two lines of (6) arise from diagrams like the one in Fig. 1. We dwell on the explicit derivation of those terms in the appendix.

In order to solve the RGE for the effective neutrino mass operators one also needs the RGE for the other couplings occurring in (6). The general RGE for an arbitrary renormalizable gauge field theory have been derived in [17,18] at the one- and two-loop levels, respectively. It is convenient to have the results of [17] specialized to the case of the multi-Higgs-doublet SM. We have found that

$$16\pi^2 \frac{d\lambda_{ijkl}}{dt} = 4 \sum_{m,n=1}^{n_H} \left(2\lambda_{ijmn}\lambda_{nmkl} + \lambda_{ijmn}\lambda_{kmnl} \right.$$

$$+ \lambda_{imnj}\lambda_{mnkl} + \lambda_{imkn}\lambda_{mjnl} + \lambda_{mjkn}\lambda_{imnl} \right)$$

$$- \left(9g^2 + 3g'^2 \right) \lambda_{ijkl} + \frac{9g^4 + 3g'^4}{8} \delta_{ij}\delta_{kl}$$

$$+ \frac{3g^2 g'^2}{4} \left(2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl} \right)$$
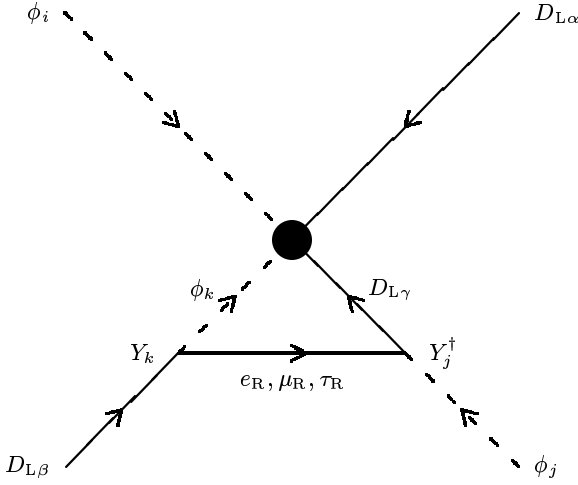
Fig. 1. A typical vertex correction in the renormalization of the operator $\mathcal{O}_{ij}$. The relevant Yukawa-coupling matrices are indicated

$$+ \sum_{m=1}^{n_H} \left(T_{mj}\lambda_{imkl} + T_{ml}\lambda_{ijkm} + T_{im}\lambda_{mjkl} + T_{km}\lambda_{ijml} \right)$$

$$-2\,\mathrm{tr}\left(Y_i Y_j^\dagger Y_k Y_l^\dagger \right), \tag{9}$$

$$16\pi^2\,\frac{\mathrm{d}Y_i}{\mathrm{d}t} \tag{10}$$

$$= \sum_{k=1}^{n_H} \left(T_{ik}Y_k + Y_k Y_k^\dagger Y_i + \frac{1}{2}\,Y_i Y_k^\dagger Y_k \right) - \frac{9g^2 + 15g'^2}{4}\,Y_i.$$

In the last line of (9) we have again left out quark contributions. It is well known that the RGE for g and g' are

$$16\pi^2\,\frac{\mathrm{d}g}{\mathrm{d}t} = \left(-\frac{22}{3} + \frac{4N}{3} + \frac{n_H}{6} \right) g^3, \tag{11}$$

$$16\pi^2\,\frac{\mathrm{d}g'}{\mathrm{d}t} = \left(\frac{20N}{9} + \frac{n_H}{6} \right) g'^3, \tag{12}$$

where $N = 3$ is the number of fermion families.

3 Application of the RGE to two particular models

3.1 The $\mathbb{Z}_2$ and D_4 models

We now apply the general RGE derived in Sect. 2 to the so-called $\mathbb{Z}_2$ [11] and D_4 [12] models – for a review see [19]. Those models predict

$$\theta_{13} = 0, \qquad \theta_{23} = \pi/4 \tag{13}$$

at the seesaw scale and are, in what regards the practical application of the RGE, identical. They both have three Higgs doublets ϕ_1, ϕ_2, and ϕ_3. Below the seesaw scale the structure of both models is dictated by the symmetries

$$\mathbb{Z}_2^{(\mathrm{aux})}: \ e_\mathrm{R} \to -e_\mathrm{R}, \ \phi_1 \to -\phi_1, \tag{14}$$

$$\mathbb{Z}_2^{(\mathrm{tr})}: \ D_{\mathrm{L}\mu} \leftrightarrow D_{\mathrm{L}\tau}, \ \mu_\mathrm{R} \leftrightarrow \tau_\mathrm{R}, \ \phi_3 \to -\phi_3. \tag{15}$$

These two symmetries hold in between the seesaw (high) scale m_R and the electroweak (low) scale m_Z. Indeed, they are broken only spontaneously, by the vacuum expectation values (VEVs) of ϕ_1^0 and ϕ_3^0, respectively, at the low scale. Because of the symmetries in (14) and (15), the Higgs potential is given by

$$V = \text{quadratic terms} + \sum_{i=1}^{3} \lambda_i \left(\phi_i^\dagger \phi_i \right)^2$$

$$+ \lambda_4 \left(\phi_1^\dagger \phi_1 \right)\left(\phi_2^\dagger \phi_2 \right) + \lambda_5 \left(\phi_1^\dagger \phi_1 \right)\left(\phi_3^\dagger \phi_3 \right)$$

$$+ \lambda_6 \left(\phi_2^\dagger \phi_2 \right)\left(\phi_3^\dagger \phi_3 \right) + \lambda_7 \left(\phi_1^\dagger \phi_2 \right)\left(\phi_2^\dagger \phi_1 \right)$$

$$+ \lambda_8 \left(\phi_1^\dagger \phi_3 \right)\left(\phi_3^\dagger \phi_1 \right) + \lambda_9 \left(\phi_2^\dagger \phi_3 \right)\left(\phi_3^\dagger \phi_2 \right) \tag{16}$$

$$+ \left[\lambda_{10} \left(\phi_1^\dagger \phi_2 \right)^2 + \lambda_{11} \left(\phi_1^\dagger \phi_3 \right)^2 + \lambda_{12} \left(\phi_2^\dagger \phi_3 \right)^2 + \mathrm{H.c.} \right],$$

where λ_{10}, λ_{11}, and λ_{12} are the only non-real quartic couplings. Comparing (16) with (1) and (2), we arrive at the identifications $\lambda_{iiii} = \lambda_i$ (for $i = 1, 2, 3$), $\lambda_{1122} = \lambda_{2211} = \lambda_4/2$, $\lambda_{1221} = \lambda_{2112} = \lambda_7/2$, $\lambda_{1212} = \lambda_{2121}^* = \lambda_{10}$, and so on.

The $\mathbb{Z}_2$ and D_4 models have three other symmetries, the family-lepton-number symmetries L_α, which are broken at the seesaw scale – softly in the $\mathbb{Z}_2$ model, spontaneously in the D_4 model. Because of the symmetries in (14) and (15), and also because of the symmetries L_α – which remain valid for the quartic couplings of the light fields below the seesaw scale – the lepton Yukawa Lagrangian is

$$\mathcal{L}_{\mathrm{Y}\ell} = -y_3 \bar{D}_{\mathrm{L}e} e_\mathrm{R}\phi_1 - y_4 \left(\bar{D}_{\mathrm{L}\mu}\mu_\mathrm{R} + \bar{D}_{\mathrm{L}\tau}\tau_\mathrm{R} \right)\phi_2$$

$$- y_5 \left(\bar{D}_{\mathrm{L}\mu}\mu_\mathrm{R} - \bar{D}_{\mathrm{L}\tau}\tau_\mathrm{R} \right)\phi_3 + \mathrm{H.c.} \tag{17}$$

(The coupling constants $y_{1,2}$ occur in the Yukawa interactions of the right-handed neutrino singlets [19] and are thus of no concern here.) Comparing (17) with (3), we see that the Yukawa-coupling matrices are

$$Y_1 = \mathrm{diag}\left(y_3^*,\ 0,\ 0 \right),$$

$$Y_2 = \mathrm{diag}\left(0,\ y_4^*,\ y_4^* \right), \tag{18}$$

$$Y_3 = \mathrm{diag}\left(0,\ y_5^*,\ -y_5^* \right).$$

Hence, from (7),

$$T_{11} = |y_3|^2, \tag{19}$$

$$T_{22} = 2|y_4|^2, \tag{20}$$

$$T_{33} = 2|y_5|^2, \tag{21}$$

and the T_{ij} with $i \neq j$ vanish, a fact which simplifies considerably the RGE in this particular case.

As emphasized before, the symmetries (14) and (15) are broken only at the electroweak scale. The validity of the symmetry $\mathbb{Z}_2^{(\mathrm{aux})}$ – which changes the sign of ϕ_1 but

does not affect the lepton doublets D_L – has an important consequence: the operators $\mathcal{O}_{12}$, $\mathcal{O}_{21}$, $\mathcal{O}_{13}$, and $\mathcal{O}_{31}$ are altogether absent. The symmetry $\mathbb{Z}_2^{(\mathrm{tr})}$, on the other hand, changes the sign of ϕ_3 simultaneously with the interchange of $D_{L\mu}$ with $D_{L\tau}$. This implies that the coupling matrices $\kappa^{(ii)}$ $(i = 1, 2, 3)$ must be of the form

$$\begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}, \tag{22}$$

while the matrices $\kappa^{(23)}$ and $\kappa^{(32)} = \kappa^{(23)\,\mathrm{T}}$ are of the form

$$\begin{pmatrix} 0 & p & -p \\ q & s & r \\ -q & -r & -s \end{pmatrix}. \tag{23}$$

3.2 The RGE for the $\mathbb{Z}_2$ and D_4 models

The renormalization group equations for the Yukawa couplings of the $\mathbb{Z}_2$ and D_4 models are

$$16\pi^2 \frac{\mathrm{d}y_3}{\mathrm{d}t} = \left(\frac{5}{2} |y_3|^2 - \frac{9g^2 + 15g'^2}{4} \right) y_3, \tag{24}$$

$$16\pi^2 \frac{\mathrm{d}y_4}{\mathrm{d}t} = \left(\frac{7}{2} |y_4|^2 + \frac{3}{2} |y_5|^2 - \frac{9g^2 + 15g'^2}{4} \right) y_4, \tag{25}$$

$$16\pi^2 \frac{\mathrm{d}y_5}{\mathrm{d}t} = \left(\frac{3}{2} |y_4|^2 + \frac{7}{2} |y_5|^2 - \frac{9g^2 + 15g'^2}{4} \right) y_5. \tag{26}$$

The RGE for the scalar-potential couplings are

$$16\pi^2 \frac{\mathrm{d}\lambda_1}{\mathrm{d}t}$$
$$= 24\lambda_1^2 + \lambda_4^2 + (\lambda_4 + \lambda_7)^2 + \lambda_5^2 + (\lambda_5 + \lambda_8)^2$$
$$+ 4 |\lambda_{10}|^2 + 4 |\lambda_{11}|^2 + \left(4 |y_3|^2 - C \right) \lambda_1 + \frac{9g^4}{8}$$
$$+ \frac{3g^2 g'^2}{4} + \frac{3g'^4}{8} - 2 |y_3|^4, \tag{27}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_2}{\mathrm{d}t}$$
$$= 24\lambda_2^2 + \lambda_4^2 + (\lambda_4 + \lambda_7)^2 + \lambda_6^2 + (\lambda_6 + \lambda_9)^2$$
$$+ 4 |\lambda_{10}|^2 + 4 |\lambda_{12}|^2 + \left(8 |y_4|^2 - C \right) \lambda_2 + \frac{9g^4}{8} + \frac{3g^2 g'^2}{4}$$
$$+ \frac{3g'^4}{8} - 4 |y_4|^4, \tag{28}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_3}{\mathrm{d}t}$$
$$= 24\lambda_3^2 + \lambda_5^2 + (\lambda_5 + \lambda_8)^2 + \lambda_6^2 + (\lambda_6 + \lambda_9)^2$$

$$+ 4 |\lambda_{11}|^2 + 4 |\lambda_{12}|^2 + \left(8 |y_5|^2 - C \right) \lambda_3 + \frac{9g^4}{8} + \frac{3g^2 g'^2}{4}$$

$$+ \frac{3g'^4}{8} - 4 |y_5|^4, \tag{29}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_4}{\mathrm{d}t}$$
$$= (\lambda_1 + \lambda_2) (12\lambda_4 + 4\lambda_7) + 4\lambda_4^2 + 2\lambda_7^2 + 4\lambda_5\lambda_6$$
$$+ 2 (\lambda_5\lambda_9 + \lambda_6\lambda_8) + 8 |\lambda_{10}|^2 \tag{30}$$
$$+ \left(2 |y_3|^2 + 4 |y_4|^2 - C \right) \lambda_4 + \frac{9g^4}{4} - \frac{3g^2 g'^2}{2} + \frac{3g'^4}{4},$$

$$16\pi^2 \frac{\mathrm{d}\lambda_5}{\mathrm{d}t}$$
$$= (\lambda_1 + \lambda_3) (12\lambda_5 + 4\lambda_8) + 4\lambda_5^2 + 2\lambda_8^2 + 4\lambda_4\lambda_6$$
$$+ 2 (\lambda_4\lambda_9 + \lambda_6\lambda_7) + 8 |\lambda_{11}|^2 \tag{31}$$
$$+ \left(2 |y_3|^2 + 4 |y_5|^2 - C \right) \lambda_5 + \frac{9g^4}{4} - \frac{3g^2 g'^2}{2} + \frac{3g'^4}{4},$$

$$16\pi^2 \frac{\mathrm{d}\lambda_6}{\mathrm{d}t}$$
$$= (\lambda_2 + \lambda_3) (12\lambda_6 + 4\lambda_9) + 4\lambda_6^2 + 2\lambda_9^2 + 4\lambda_4\lambda_5$$
$$+ 2 (\lambda_4\lambda_8 + \lambda_5\lambda_7) + 8 |\lambda_{12}|^2$$
$$+ \left(4 |y_4|^2 + 4 |y_5|^2 - C \right) \lambda_6$$
$$+ \frac{9g^4}{4} - \frac{3g^2 g'^2}{2} + \frac{3g'^4}{4} - 8 |y_4 y_5|^2, \tag{32}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_7}{\mathrm{d}t}$$
$$= \left(4\lambda_1 + 4\lambda_2 + 8\lambda_4 + 4\lambda_7 + 2 |y_3|^2 + 4 |y_4|^2 - C \right) \lambda_7$$
$$+ 2\lambda_8\lambda_9 + 32 |\lambda_{10}|^2 + 3g^2 g'^2, \tag{33}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_8}{\mathrm{d}t}$$
$$= \left(4\lambda_1 + 4\lambda_3 + 8\lambda_5 + 4\lambda_8 + 2 |y_3|^2 + 4 |y_5|^2 - C \right) \lambda_8$$
$$+ 2\lambda_7\lambda_9 + 32 |\lambda_{11}|^2 + 3g^2 g'^2, \tag{34}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_9}{\mathrm{d}t}$$
$$= \left(4\lambda_2 + 4\lambda_3 + 8\lambda_6 + 4\lambda_9 + 4 |y_4|^2 + 4 |y_5|^2 - C \right) \lambda_9$$
$$+ 2\lambda_7\lambda_8 + 32 |\lambda_{12}|^2 + 3g^2 g'^2 - 8 |y_4 y_5|^2, \tag{35}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_{10}}{\mathrm{d}t}$$
$$= (4\lambda_1 + 4\lambda_2 + 8\lambda_4 + 12\lambda_7$$
$$+ 2 |y_3|^2 + 4 |y_4|^2 - C \right) \lambda_{10} + 4\lambda_{11}\lambda_{12}^*, \tag{36}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_{11}}{\mathrm{d}t}$$
$$= \left(4\lambda_1 + 4\lambda_3 + 8\lambda_5 + 12\lambda_8 + 2 |y_3|^2 \right.$$

$$+4\left|y_5\right|^2 - C\Big)\lambda_{11} + 4\lambda_{10}\lambda_{12}, \tag{37}$$

$$16\pi^2 \frac{\mathrm{d}\lambda_{12}}{\mathrm{d}t}$$
$$= \Big(4\lambda_2 + 4\lambda_3 + 8\lambda_6 + 12\lambda_9 + 4\left|y_4\right|^2$$
$$+ 4\left|y_5\right|^2 - C\Big)\lambda_{12} + 4\lambda_{10}^*\lambda_{11} - 4y_4^{*2}y_5^2, \tag{38}$$

where
$$C := 9g^2 + 3g'^2. \tag{39}$$

The reason why no fourth-order terms in the Yukawa couplings and in the gauge couplings appear in the RGE for λ_{10} and λ_{11} is that the condition $\lambda_{10} = \lambda_{11} = 0$ may be enforced through an additional $U(1)$ symmetry: $\phi_1 \to \mathrm{e}^{\mathrm{i}\alpha}\phi_1$, $e_{\mathrm{R}} \to \mathrm{e}^{-\mathrm{i}\alpha}e_{\mathrm{R}}$, where $\alpha \in \mathbb{R}$.

We next write down the RGE for the coupling matrices of the effective neutrino mass operators. They are

$$16\pi^2 \frac{\mathrm{d}\kappa^{(11)}}{\mathrm{d}t}$$
$$= \left(-3g^2 + 4\lambda_1 + 2\left|y_3\right|^2\right)\kappa^{(11)} + 4\lambda_{10}^*\kappa^{(22)} + 4\lambda_{11}^*\kappa^{(33)}$$
$$+ \left\{\kappa^{(11)}, P - 2P_1\right\}, \tag{40}$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(22)}}{\mathrm{d}t}$$
$$= \left(-3g^2 + 4\lambda_2 + 4\left|y_4\right|^2\right)\kappa^{(22)} + 4\lambda_{10}\kappa^{(11)} + 4\lambda_{12}^*\kappa^{(33)}$$
$$+ \left\{\kappa^{(22)}, P - 2P_2\right\} - 2\left(\kappa^{(23)}P_{23} + P_{23}\kappa^{(32)}\right), \tag{41}$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(33)}}{\mathrm{d}t}$$
$$= \left(-3g^2 + 4\lambda_3 + 4\left|y_5\right|^2\right)\kappa^{(33)} + 4\lambda_{11}\kappa^{(11)} + 4\lambda_{12}\kappa^{(22)}$$
$$+ \left\{\kappa^{(33)}, P - 2P_3\right\} - 2\left(\kappa^{(32)}P_{32} + P_{32}\kappa^{(23)}\right), \tag{42}$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(23)}}{\mathrm{d}t}$$
$$= \left(-3g^2 + 2\lambda_6 + 2\left|y_4\right|^2 + 2\left|y_5\right|^2\right)\kappa^{(23)} + 2\lambda_9\kappa^{(32)}$$
$$+ \left\{\kappa^{(23)}, P\right\} - 4\kappa^{(22)}P_{32} + 2P_{32}\kappa^{(22)}$$
$$- 4P_{23}\kappa^{(33)} + 2\kappa^{(33)}P_{23} + 2\left[\kappa^{(23)}, P_2 - P_3\right]$$
$$- 2\left(\kappa^{(32)}P_3 + P_2\kappa^{(32)}\right), \tag{43}$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(32)}}{\mathrm{d}t}$$
$$= \left(-3g^2 + 2\lambda_6 + 2\left|y_4\right|^2 + 2\left|y_5\right|^2\right)\kappa^{(32)} + 2\lambda_9\kappa^{(23)}$$
$$+ \left\{\kappa^{(32)}, P\right\} - 4\kappa^{(33)}P_{23} + 2P_{23}\kappa^{(33)} - 4P_{32}\kappa^{(22)}$$
$$+ 2\kappa^{(22)}P_{32} + 2\left[\kappa^{(32)}, P_3 - P_2\right]$$

$$-2\left(\kappa^{(23)}P_2 + P_3\kappa^{(23)}\right), \tag{44}$$

where $\{R, S\}$ and $[R, S]$ denote the anticommutator and the commutator, respectively, of the matrices R and S. Moreover, we have defined

$$P_1 := \mathrm{diag}\left(\left|y_3\right|^2,\ 0,\ 0\right), \tag{45}$$

$$P_2 := \mathrm{diag}\left(0,\ \left|y_4\right|^2,\ \left|y_4\right|^2\right), \tag{46}$$

$$P_3 := \mathrm{diag}\left(0,\ \left|y_5\right|^2,\ \left|y_5\right|^2\right), \tag{47}$$

$$P_{23} := \mathrm{diag}\left(0,\ y_4 y_5^*,\ -y_4 y_5^*\right), \tag{48}$$

$$P_{32} := \mathrm{diag}\left(0,\ y_4^* y_5,\ -y_4^* y_5\right). \tag{49}$$

Notice that the matrix P defined in (8) is now given by $(P_1 + P_2 + P_3)/2$.

4 Predictions of the $\mathbb{Z}_2$ and D_4 models

The Lagrangian of neutrino Majorana masses is

$$\mathcal{L}_{\mathrm{Majorana}} = \frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau} \nu_{\mathrm{L}\alpha}^{\mathrm{T}} C^{-1}\left(\mathcal{M}_\nu\right)_{\alpha\beta}\nu_{\mathrm{L}\beta} + \mathrm{H.c.}, \tag{50}$$

where $\mathcal{M}_\nu = \mathcal{M}_\nu^{\mathrm{T}}$. Taking $b = d = 2$ in (4), it is clear that

$$\mathcal{O}_{ij} = \sum_{\alpha,\beta=e,\mu,\tau}\kappa_{\alpha\beta}^{(ij)}\phi_i^0\phi_j^0\nu_{\mathrm{L}\alpha}^{\mathrm{T}}C^{-1}\nu_{\mathrm{L}\beta} + \ldots. \tag{51}$$

Therefore, if we denote the VEV of ϕ_i^0 by v_i, then the neutrino Majorana mass matrix $\mathcal{M}_\nu$ is given by

$$\frac{1}{2}\mathcal{M}_\nu = \sum_{i=1}^{3} v_i^2\kappa^{(ii)} + v_2 v_3\left[\kappa^{(23)} + \kappa^{(32)}\right], \tag{52}$$

since $\kappa^{(12)} = \kappa^{(21)} = \kappa^{(13)} = \kappa^{(31)} = 0$ in the $\mathbb{Z}_2$ and D_4 models. This is valid at all scales μ.

In general one may write [20]

$$\mathcal{M}_\nu = \begin{pmatrix} X & A(1+\epsilon) & A(1-\epsilon) \\ A(1+\epsilon) & B(1+\epsilon') & C \\ A(1-\epsilon) & C & B(1-\epsilon') \end{pmatrix}. \tag{53}$$

We already know that the form of the flavor coupling matrices $\kappa^{(ii)}$ is described by (22), while $\kappa^{(23)} = \kappa^{(32)\mathrm{T}}$ is described by (23). Therefore,

$$\sum_{i=1}^{3} 2v_i^2\kappa^{(ii)} = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix}, \tag{54}$$

while

$$\kappa^{(c)} := \kappa^{(23)} + \kappa^{(32)} = \begin{pmatrix} 0 & c_1 & -c_1 \\ c_1 & c_2 & 0 \\ -c_1 & 0 & -c_2 \end{pmatrix}, \tag{55}$$

with

$$\epsilon A = 2v_2 v_3 c_1, \tag{56}$$

$$\epsilon' B = 2v_2 v_3 c_2. \tag{57}$$

Once again, this is valid at any scale μ.

In both the $\mathbb{Z}_2$ and D_4 models, the symmetry $\mathbb{Z}_2^{(\text{aux})}$ inverts the signs of the right-handed-neutrino fields which are present above the seesaw scale. Hence, only the doublet ϕ_1 has Yukawa couplings to those fields, above the high scale. This implies that $\mathcal{M}_\nu(t_0)$, where $t_0 := \ln m_{\mathrm{R}}$, originates solely from the VEV of ϕ_1^0 [11,12]. Therefore, at the seesaw scale [14]

$$\kappa^{(11)}(t_0) = \mathcal{M}_\nu(t_0) / \left(2v_1^2\right),$$

$$\kappa^{(ij)}(t_0) = 0 \text{ for all other } (ij). \tag{58}$$

We conclude that $\mathcal{M}_\nu(t_0)$ has the same form as $\kappa^{(11)}$, i.e. $\mathcal{M}_\nu(t_0)$ is of the form (22). Clearly then, $(0, 1, -1)^{\mathrm{T}}$ is an eigenvector of $\mathcal{M}_\nu(t_0)$ and therefore, at the seesaw scale, the predictions (13) hold.

At any other scale, though, the matrix $\kappa^{(c)}$ in (55) is not zero. Thus, for any $t < t_0$, $\mathcal{M}_\nu(t)$ is not $\mu \leftrightarrow \tau$-symmetric. This fact renders the predictions (13) *inexact for any scale other than the seesaw scale*. In [20] it has been shown that, if one assumes the parameters ϵ and ϵ' in (53) to be small, then, to first order in those parameters, one has, instead of (13),

$$U_{e3} = \frac{s_{12} c_{12}}{m_3^2 - m_2^2} \left(\bar{\epsilon} s_{12}^2 \hat{m}_2^* + \bar{\epsilon}^* s_{12}^2 m_3 - \bar{\epsilon}' \hat{m}_2^* - \bar{\epsilon}'^* m_3\right)$$

$$+ \frac{s_{12} c_{12}}{m_3^2 - m_1^2} \left(\bar{\epsilon} c_{12}^2 \hat{m}_1^* + \bar{\epsilon}^* c_{12}^2 m_3 + \bar{\epsilon}' \hat{m}_1^* + \bar{\epsilon}'^* m_3\right), \tag{59}$$

$$\cos 2\theta_{23} = \mathrm{Re}\left[\frac{2c_{12}^2}{m_3^2 - m_2^2} \left(\bar{\epsilon} s_{12}^2 - \bar{\epsilon}'\right)(\hat{m}_2 + m_3)^*\right.$$

$$\left. - \frac{2s_{12}^2}{m_3^2 - m_1^2}\left(\bar{\epsilon} c_{12}^2 + \bar{\epsilon}'\right)(\hat{m}_1 + m_3)^*\right], \tag{60}$$

where

$$\bar{\epsilon} := (\hat{m}_1 - \hat{m}_2)\,\epsilon, \tag{61}$$

$$\bar{\epsilon}' := \frac{\hat{m}_1 s_{12}^2 + \hat{m}_2 c_{12}^2 + m_3}{2}\,\epsilon', \tag{62}$$

and

$$\hat{m}_1 = m_1 \mathrm{e}^{-2\mathrm{i}\rho}, \tag{63}$$

$$\hat{m}_2 = m_2 \mathrm{e}^{-2\mathrm{i}\sigma}. \tag{64}$$

Here, $m_1, m_2,$ and m_3 are the (real, non-negative) neutrino masses, while ρ and σ are Majorana phases. In (59)–(62), s_{12} and c_{12} are the sine and cosine, respectively, of the solar mixing angle. We are using the standard parametrization [9,10] for the lepton mixing matrix U; that parametrization fixes the significance of the phase of (59).

We see in (59) and (60) that the deviation from the predictions (13) is numerically of the same order as the (small) parameters ϵ and ϵ', except in the case of quasi-degenerate neutrinos with common mass m_0; in that case, an enhancement factor $m_0^2/\Delta m_{\mathrm{atm}}^2$ appears, where

$$\Delta m_{\mathrm{atm}}^2 = \left|m_3^2 - m_2^2\right| \approx \left|m_3^2 - m_1^2\right|$$

is the atmospheric mass-squared difference.

5 An approximate solution of the RGE

In this section we assume the Yukawa couplings y_3, y_4, and y_5 to be very small. This assumption is reasonable when one considers the $\mathbb{Z}_2$ or D_4 models with their minimal content of three Higgs doublets, since in that case $\sum_{i=1}^3 |v_i|^2$ must be equal to $(174\,\mathrm{GeV})^2$, and therefore all the $|v_i|$ are in principle much larger than the charged-lepton masses. We estimate, to first order in the Yukawa couplings squared, the deviation of $\mathcal{M}_\nu(t_1)$, where $t_1 := \ln m_Z$, from the $\mu \leftrightarrow \tau$-symmetric form. We thus find out the likely magnitudes of U_{e3} and $\cos 2\theta_{23}$ at the electroweak scale.

The starting point is (40)–(44). We drop all the Yukawa couplings from (40)–(42); in (43) and (44), on the other hand, we keep the terms with P_{23} and P_{32}, since it is those terms which induce corrections to the $\mu \leftrightarrow \tau$-symmetric form of $\mathcal{M}_\nu(t_0)$. We thus obtain

$$16\pi^2 \frac{\mathrm{d}\kappa^{(11)}}{\mathrm{d}t} \tag{65}$$

$$\approx \left(-3g^2 + 4\lambda_1\right)\kappa^{(11)} + 4\lambda_{10}^* \kappa^{(22)} + 4\lambda_{11}^* \kappa^{(33)},$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(22)}}{\mathrm{d}t} \tag{66}$$

$$\approx \left(-3g^2 + 4\lambda_2\right)\kappa^{(22)} + 4\lambda_{10}\kappa^{(11)} + 4\lambda_{12}^* \kappa^{(33)},$$

$$16\pi^2 \frac{\mathrm{d}\kappa^{(33)}}{\mathrm{d}t} \tag{67}$$

$$\approx \left(-3g^2 + 4\lambda_3\right)\kappa^{(33)} + 4\lambda_{11}\kappa^{(11)} + 4\lambda_{12}\kappa^{(22)},$$

and

$$16\pi^2 \frac{\mathrm{d}\kappa^{(c)}}{\mathrm{d}t} = \left(-3g^2 + 2\lambda_6 + 2\lambda_9\right)\kappa^{(c)} \tag{68}$$

$$- 2\left\{P_{32}, \kappa^{(22)}\right\} - 2\left\{P_{23}, \kappa^{(33)}\right\},$$

where $\kappa^{(c)} = \kappa^{(23)} + \kappa^{(32)}$ as in (55). Using (48) and (49), we obtain from (69) that

$$16\pi^2 \frac{\mathrm{d}c_1}{\mathrm{d}t} \tag{69}$$

$$= \left(-3g^2 + 2\lambda_6 + 2\lambda_9\right)c_1 - 2\left[y_4^* y_5 \kappa_{12}^{(22)} + y_4 y_5^* \kappa_{12}^{(33)}\right],$$

$$16\pi^2 \frac{\mathrm{d}c_2}{\mathrm{d}t} \tag{70}$$

$$= \left(-3g^2 + 2\lambda_6 + 2\lambda_9\right)c_2 - 4\left[y_4^* y_5 \kappa_{22}^{(22)} + y_4 y_5^* \kappa_{22}^{(33)}\right].$$

Formally we can write the solution of the coupled differential equations (65)–(67) as

$$\kappa^{(ii)}(t) = \sum_{j=1}^{3} T(t, t')_{ij} \, \kappa^{(jj)}(t'). \tag{71}$$

All the matrix elements of $\kappa^{(ii)}$ evolve according to the same operator $T(t, t')$. Since at the scale t_0 only $\kappa^{(11)}$ is non-vanishing,

$$\kappa^{(ii)}(t) = T(t, t_0)_{i1} \, \kappa^{(11)}(t_0). \tag{72}$$

Formally, the solutions to (69) and (70) are easily written down. Defining

$$S_c(t) := \exp\left[\frac{1}{16\pi^2}\int_{t_0}^{t} dt' \left(-3g^2 + 2\lambda_6 + 2\lambda_9\right)(t')\right] \tag{73}$$

and taking into account that $c_1(t_0) = c_2(t_0) = 0$, one has

$$c_1(t) = -\frac{1}{8\pi^2} S_c(t) \int_{t_0}^{t} dt' \, S_c^{-1}(t')$$
$$\times \left[y_4^* y_5 \kappa_{12}^{(22)} + y_4 y_5^* \kappa_{12}^{(33)}\right](t')$$
$$= -\frac{\kappa_{12}^{(11)}(t_0)}{8\pi^2} S_c(t) \int_{t_0}^{t} dt' \, S_c^{-1}(t') \tag{74}$$
$$\times \left[\nu(t') T(t', t_0)_{21} + \nu^*(t') T(t', t_0)_{31}\right],$$

$$c_2(t) = -\frac{1}{4\pi^2} S_c(t) \int_{t_0}^{t} dt' \, S_c^{-1}(t')$$
$$\times \left[y_4^* y_5 \kappa_{22}^{(22)} + y_4 y_5^* \kappa_{22}^{(33)}\right](t')$$
$$= -\frac{\kappa_{22}^{(11)}(t_0)}{4\pi^2} S_c(t) \int_{t_0}^{t} dt' \, S_c^{-1}(t') \tag{75}$$
$$\times \left[\nu(t') T(t', t_0)_{21} + \nu^*(t') T(t', t_0)_{31}\right],$$

where we have used (72) and defined $\nu := y_4^* y_5$.
We now make use of (54) and (56) to write

$$\epsilon(t_1) = \frac{v_2 v_3 c_1(t_1)}{\sum_{i=1}^{3} v_i^2 \kappa_{12}^{(ii)}(t_1)}$$
$$= \frac{v_2 v_3 c_1(t_1)}{\sum_{i=1}^{3} v_i^2 T(t_1, t_0)_{i1}} \frac{1}{\kappa_{12}^{(11)}(t_0)}. \tag{76}$$

Similarly,

$$\epsilon'(t_1) = \frac{v_2 v_3 c_2(t_1)}{\sum_{i=1}^{3} v_i^2 T(t_1, t_0)_{i1}} \frac{1}{\kappa_{22}^{(11)}(t_0)}. \tag{77}$$

Putting (74)–(77) together, we conclude that

$$\epsilon(t_1) = -\frac{v_2 v_3}{8\pi^2 \sum_{i=1}^{3} v_i^2 T(t_1, t_0)_{i1}} S_c(t_1) \int_{t_0}^{t_1} dt' \, S_c^{-1}(t')$$
$$\times \left[\nu(t') T(t', t_0)_{21} + \nu^*(t') T(t', t_0)_{31}\right], \tag{78}$$

while $\epsilon'(t_1) = 2\epsilon(t_1)$. From (59) and (60) one can derive that

$$\epsilon' = 2\epsilon \Rightarrow$$
$$\begin{cases} U_{e3} = \\ \quad 2m_3 c_{12} s_{12} \left(\dfrac{\hat{m}_1^* + m_3}{m_3^2 - m_1^2} + \dfrac{\hat{m}_2^* + m_3}{m_2^2 - m_3^2}\right) \mathrm{Re}\,\epsilon, \\ \cos 2\theta_{23} = \\ \quad 2\left(s_{12}^2 \dfrac{|\hat{m}_1 + m_3|^2}{m_1^2 - m_3^2} + c_{12}^2 \dfrac{|\hat{m}_2 + m_3|^2}{m_2^2 - m_3^2}\right) \mathrm{Re}\,\epsilon. \end{cases} \tag{79}$$

Now we want to estimate the maximum possible order of magnitude of $\epsilon(t_1)$ by using (78). The length of the integration interval of t' is $t_0 - t_1 = \ln(m_R/m_Z) \sim 10$. The functions $S_c(t)$ and $S_c^{-1}(t')$ are of order 1 since, in (73), $\lambda_6/(16\pi^2)$ and $\lambda_9/(16\pi^2)$ are necessarily small. The functions $T(t', t_0)_{21}$ and $T(t', t_0)_{31}$ are governed by λ_{10} and λ_{11}; in any case, we expect $T(t', t_0)_{21}/T(t_1, t_0)_{i1}$ and $T(t', t_0)_{31}/T(t_1, t_0)_{i1}$ to be $\lesssim 1$. Similarly, $v_2 v_3/v_i^2$ should not be larger than 1. We conclude that

$$|\epsilon(t_1)| \sim \frac{10|\nu|}{8\pi^2} \sim \frac{|y_4 y_5|}{10}. \tag{80}$$

Even if we allow for rather small VEVs, $|y_{4,5}|$ cannot be larger than 0.1. We thus have the generous upper bound $|\epsilon(t_1)| \lesssim 10^{-3}$.

Equation (79) tells us that the only chance to have a non-negligible U_{e3} is in the case of a degenerate neutrino spectrum. Let us consider the extreme case of a common mass $m_0 = 0.3\,\mathrm{eV}$ [21]. Since $\Delta m_{\mathrm{atm}}^2 \simeq 2 \times 10^{-3}\,\mathrm{eV}^2$, in that case we have $|U_{e3}| \simeq 100\,|\mathrm{Re}\,\epsilon| \lesssim 0.1$. Here we have used Majorana phases $\rho = \sigma = 0$ for simplicity. That choice of m_0 is indeed extreme; if take $m_0 = 0.1\,\mathrm{eV}$ instead, then the upper bound becomes one order of magnitude smaller, due to the quadratic dependence of $|U_{e3}|$ on m_0 in the case of a degenerate neutrino spectrum. In any case, we expect $|U_{e3}|$ and $|\cos 2\theta_{23}|$ to be no larger than 0.1 in our model, but most likely they are two or more orders of magnitude smaller.

6 Summary

In this paper we have computed the RGE for the dimension-five neutrino mass operators in the multi-Higgs-doublet SM. Thus, the main result of this paper is (6), which describes the evolution of the coupling matrices of the mass operators in the SM with an arbitrary number of Higgs doublets. We have argued in favor of the usefulness of (6) by citing models for lepton mixing which have been constructed in the framework of the multi-Higgs-doublet SM.

As an application of our RGE we have considered the $\mathbb{Z}_2$ model of [11] and the D_4 model of [12], which – from the field-theoretical point of view – are identical below the seesaw scale. The predictions (13) of those models hold at the seesaw scale and we have used the RGE to estimate the corrections to those predictions which appear due to

the evolution of the coupling matrices of the dimension-five neutrino mass operators down to the electroweak scale. We have found that those corrections are in general negligible, with the possible exception of a degenerate neutrino mass spectrum with a rather large common mass $m_0 \gtrsim 0.2\,\text{eV}$. In that case, s_{13}^2 could be as large as 0.01 and be within the sensitivity of the planned long-baseline neutrino experiments [9]. On the other hand, even in the degenerate neutrino case the deviation of θ_{23} from $\pi/4$ will be hard to uncover in those experiments [22], since they will be sensitive to the parameter $\sin^2 2\theta_{23} = 1 - \cos^2 2\theta_{23}$ and we have estimated $\cos^2 2\theta_{23} \lesssim 0.01$ in our models. Thus, with respect to the experiments presently envisaged the models discussed here have the following properties: should a non-zero s_{13}^2 be discovered, then the neutrino mass spectrum must be degenerate, while deviations from $\sin^2 2\theta_{23} = 1$ should be invisible.

A Vertex corrections to the neutrino mass operators

The last two lines of (6) originate in vertex corrections of the type displayed in Fig. 1. In this appendix we show how we have arrived at those two lines. Perturbation theory yields the expression

$$
\frac{(-\mathrm{i})^2}{2!}\,\mathbf{T}\Bigg[\sum_{m,n=1}^{n_H} \sum_{a,b,c,d=1}^{2} \Big(D_{\mathrm{L}a}^{\mathrm{T}}\kappa^{(mn)}C^{-1}D_{\mathrm{L}c}\,\varepsilon^{ab}\phi_{mb}\,\varepsilon^{cd}\phi_{nd}\Big)_x
$$

$$
\times\, 2\int \mathrm{d}^d x_1 \sum_{k=1}^{n_H}\sum_{e=1}^{2}\Big(\bar{D}_{\mathrm{L}e}Y_k^{\dagger}\phi_{ke}\ell_{\mathrm{R}}\Big)_{x_1}
\tag{A1}
$$

$$
\times \int \mathrm{d}^d x_2 \sum_{l=1}^{n_H}\sum_{f=1}^{2}\Big(\bar{\ell}_{\mathrm{R}}\phi_{lf}^{\dagger}Y_l D_{\mathrm{L}f}\Big)_{x_2}\Bigg],
$$

where d is the dimension of space-time and x, x_1, x_2 are space-time points. We have left out the flavor indices in (A1). The gauge-$SU(2)$ indices are $a, b, \ldots, f$. The symbol $\mathbf{T}$ denotes time ordering. When computing (A1), the field ℓ_{R} must be contracted with $\bar{\ell}_{\mathrm{R}}$. As for $\bar{D}_{\mathrm{L}e}$, it may be contracted either with $D_{\mathrm{L}a}$ or with $D_{\mathrm{L}c}$; it is easy to see that both possibilities yield the same contribution to the RGE of $\kappa^{(ij)}$ – this fact explains the factor 2 in the fourth line of (6). In the following we compute explicitly the case where $\bar{D}_{\mathrm{L}e}$ is contracted with $D_{\mathrm{L}c}$.

For the contraction of $\phi_{lf}^{\dagger}$ there are also two possibilities: one may contract it either with ϕ_{mb} or with ϕ_{nd}. We consider the second possibility first. We use dimensional regularization with minimal subtraction. In the evaluation of (A1) we only need the pole terms in $\epsilon = 4 - d$. The computation is straightforward and we arrive at

$$
\frac{1}{16\pi^2\epsilon}
\tag{A2}
$$

$$
\times \sum_{k,m,n=1}^{n_H}\sum_{a,b,c,d=1}^{2} D_{\mathrm{L}a}^{\mathrm{T}}C^{-1}\Big[\kappa^{(mn)}Y_k^{\dagger}Y_n\Big]D_{\mathrm{L}d}\,\varepsilon^{ab}\phi_{mb}\,\varepsilon^{dc}\phi_{kc},
$$

where all the fields are now meant to be at the same space–time point. The minus sign from the factor $(-\mathrm{i})^2$ in (A1) has been removed through the interchange of the indices c and d in ε^{cd}. If, instead, we contract $\phi_{lf}^{\dagger}$ with ϕ_{mb}, then we get

$$
-\frac{1}{16\pi^2\epsilon}
\tag{A3}
$$

$$
\times \sum_{k,m,n=1}^{n_H}\sum_{a,b,c,d=1}^{2} D_{\mathrm{L}a}^{\mathrm{T}}C^{-1}\Big[\kappa^{(mn)}Y_k^{\dagger}Y_m\Big]D_{\mathrm{L}b}\,\varepsilon^{ab}\varepsilon^{cd}\phi_{kc}\phi_{nd}.
$$

The $SU(2)$ structure of (A3) is different from the one in (4). Therefore, we need to apply the identity

$$
\varepsilon^{ab}\varepsilon^{cd} + \varepsilon^{ac}\varepsilon^{db} + \varepsilon^{ad}\varepsilon^{bc} = 0
\tag{A4}
$$

to (A3), obtaining

$$
\frac{1}{16\pi^2\epsilon}\sum_{k,m,n=1}^{n_H}\sum_{a,b,c,d=1}^{2} D_{\mathrm{L}a}^{\mathrm{T}}C^{-1}\Big[\kappa^{(mn)}Y_k^{\dagger}Y_m\Big]D_{\mathrm{L}b}
$$

$$
\times \Big(-\varepsilon^{ac}\phi_{kc}\,\varepsilon^{bd}\phi_{nd} + \varepsilon^{ad}\phi_{nd}\,\varepsilon^{bc}\phi_{kc}\Big).
\tag{A5}
$$

The terms in the last two lines of (6) are obtained from (A2) and (A5) as follows. Firstly, one substitutes the factor $1/\epsilon$ by -1 [7]. Secondly, the indices k and m – in (A2) – or k and n – in (A5) – of the scalar doublets must be replaced by i and j; the contribution to the beta function of $\kappa^{(ij)}$ is then given by the flavor matrix in between the lepton doublets. If, in each expression, the first Higgs doublet is labeled i and the second one is labeled j, then one obtains the terms in the fourth line of (6): $-\sum_n \kappa^{(in)}Y_j^{\dagger}Y_n$ from (A2), $\sum_m \Big[\kappa^{(mj)}Y_i^{\dagger}Y_m - \sum_m \kappa^{(mi)}Y_j^{\dagger}Y_m\Big]$ from (A5). Reversing the role of i and j leads to the terms in the last line of (6).

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